Optimizing microalgal productivity in raceway ponds through a controlled mixing device

Olivier Bernard, Liu-Di LU, Julien Salomon

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Introduction

- Motivation: High biotechnological potential, e.g. colorants, antioxydants, cosmetics, pharmaceuticals, food complements, green energy, etc.
- Reactors

Figure: Raceways

Han model [2]

- A: open and ready to harvest a photon,
  B: closed while processing the absorbed photon energy,
  C: inhibited if several photons have been absorbed simultaneously.
- $$\begin{cases} \dot{A} = -\sigma I A + \frac{B}{\tau}, \\ \dot{B} = \sigma I A - \frac{B}{\tau} + k_r C - k_d \sigma I B, \\ \dot{C} = -k_r C + k_d \sigma I B. \end{cases} \quad (1)$$
- A, B, C are the relative frequencies of the three possible states with $A + B + C = 1$.
- Following [3], (1) can be reduced to:
  $$\dot{C} = -\alpha(I) C + \beta(I), \quad (2)$$
  where $\alpha(I) = \beta(I) + k_r$ and $\beta(I) = k_d \tau \frac{(\sigma I)^2}{\tau \sigma I + 1}$.
- The net specific growth rate:
  $$\mu(C, I) := -\gamma(I) C + \zeta(I),$$
  where $\zeta(I) = \gamma(I) - R$ and $\gamma(I) = \frac{k \sigma I}{\tau \sigma I + 1}$.

Light intensity

- Beer-Lambert law is given by:

$$I(z) = I_s \exp(\varepsilon z),$$

- The average net specific growth rate over the domain is defined by

$$\bar{\mu} := \frac{1}{T}\int_0^T \frac{1}{h}\int_{-h}^0 \mu\big(C(t,z), I(z)\big)\mathrm{d}z\mathrm{d}t,$$

- Vertical discretization with N layers

$$z_n = -\frac{n-\frac{1}{2}}{N}h, \quad n = 1, \dots, N.$$

- Semi-discrete average net specific growth rate can be defined by

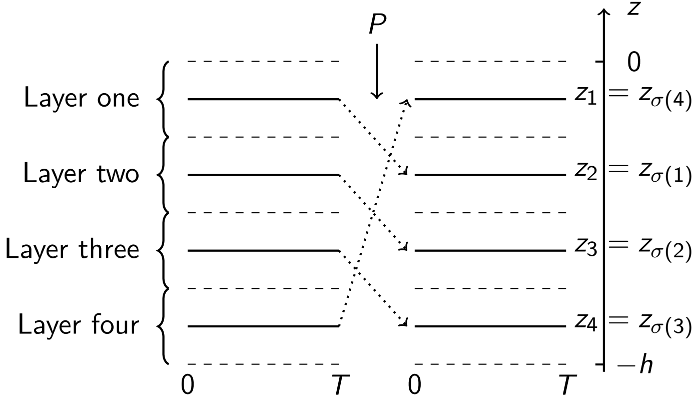
$$\bar{\mu}_N := \frac{1}{T}\int_0^T \frac{1}{N}\sum_{n=1}^{N} \mu(C_n(t), I_n)\mathrm{d}t. \tag{3}$$

Explicit computation

- For a given initial vector of states $(C_n(0))_{n=1}^N$, the solution of (2) is given by
$$C(t) = D(t)C(0) + V(t), \quad t \in [0, T], \tag{4}$$
where $D(t)$ is a diagonal matrix with $D_{nn}(t) = e^{-\alpha(I_n)t}$ and $V(t)$ is a vector with $V_n(t) = \frac{\beta(I_n)}{\alpha(I_n)}(1 - e^{-\alpha(I_n)t})$.
- (3) can also be computed explicitly, which gives
$$\bar{\mu}_N = \frac{1}{N}\frac{1}{T}\Big(\langle \Gamma, C(0)\rangle + \langle 1, Z\rangle\Big), \tag{5}$$
where 1 is a vector of size N whose coefficients equal 1, and Γ, Z are two vectors with $\Gamma_n = \frac{\gamma(I_n)}{\alpha(I_n)}(e^{-\alpha(I_n)T} - 1)$ and
$Z_n = \frac{\gamma(I_n)}{\alpha(I_n)}\frac{\beta(I_n)}{\alpha(I_n)}(1 - e^{-\alpha(I_n)T}) - \frac{\gamma(I_n)\beta(I_n)}{\alpha(I_n)}T + \zeta(I_n)T$.
- For simplicity of notations, we write hereafter D, V instead of $D(T), V(T)$.

Mixing devices

- An ideal rearrangement of trajectories: at each new lap, the algae at depth $z_i(0)$ are entirely transferred into the position $z_j(0)$ when passing through the mixing device.
- We denote by $\mathcal{P}$ the set of permutation matrices of size $N \times N$ and by $\mathfrak{S}_N$ the associated set of permutations of N elements.



Denote by $C^k(0)$ the photo-inhibition state of the algae which has just passed the mixing device P after k laps. The initial state of the system $C^0(0) := C(0)$ is assumed to be known. By definition of P, we have

$$C^{k+1}(0) = PC^k(T) = P(DC^k(0) + V).$$

Assume that the state C is KT-periodic in the sense that after K times of passing the device (P), i.e. $C^K(0) = C(0)$.

Lemma

Given $k \in \mathbb{N}$, the matrix $\mathcal{I}_N - (PD)^k$ is invertible.

Theorem

For all $k \in \mathbb{N}$

$$C^k(0) = (\mathcal{I}_N - PD)^{-1} PV. \tag{6}$$

As a consequence, the sequence $\left(C^k(0)\right)_{k\in\mathbb{N}}$ is constant.

- The average net specific growth rate for K laps of the raceway pond is then defined by

$$\bar{\mu}_N^K := \frac{1}{K} \sum_{k=0}^{K-1} \frac{1}{N} \frac{1}{T} \Big(\langle \Gamma, C^k(0) \rangle + \langle 1, Z \rangle \Big).$$

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- Average growth rate

$$\bar{\mu}_N = \frac{1}{N}\frac{1}{T}\Big(\langle \Gamma, C(0)\rangle + \langle 1, Z\rangle\Big) = \frac{1}{N}\frac{1}{T}\Big(\langle \Gamma, (\mathcal{I}_N - PD)^{-1}PV\rangle + \langle 1, Z\rangle\Big).$$

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- Find a permutation matrix $P_{\max}$ solving the maximization problem:

$$\max_{P\in\mathcal{P}}\langle\Gamma, (\mathcal{I}_N - PD)^{-1}PV\rangle. \tag{7}$$

- For realistic cases, e.g., large values of N, Problem (7) cannot be tackled in practice.
- consider the following expansion

$$\begin{aligned}\langle \Gamma, (\mathcal{I}_N - PD)^{-1} PV \rangle = &\sum_{m=0}^{+\infty} \langle \Gamma, (PD)^m PV \rangle \\ = &\langle \Gamma, PV \rangle + \sum_{m=1}^{+\infty} \langle \Gamma, (PD)^m PV \rangle.\end{aligned}$$

- Find a permutation matrix P_+ solving the maximization problem:

$$\max_{P \in \mathcal{P}} \langle \Gamma, PV \rangle. \tag{8}$$

Lemma

Let $u, v \in \mathbb{R}^N$, with $u_1 \leq \ldots \leq u_N$, $\sigma_+ \in \mathfrak{S}_N$ such that $v_{\sigma_+(1)} \leq \ldots \leq v_{\sigma_+(N)}$ and $P_+ \in \mathcal{P}$ the corresponding matrix. Then

$$P_+ \in \mathrm{argmax}_{P \in \mathcal{P}} \langle u, Pv \rangle.$$

Remark

Once Γ *and* V *are given, the optimal solution* P_+ *of* $\langle \Gamma, PV \rangle$ *can be determined explicitly as the matrix corresponding to the permutation which associates the largest element of* Γ *with the largest element of* V*, the second largest element with the second largest, and so on.*

Remark

The generalized optimization problems associated with original problem and approximated problem have been studied in [1], where we have proved that in some cases, these two problems have the same solution.

Numerical settings

- Set $N = 11$ the number of layers, meaning that we test numerically $N!$ (i.e. 39916800) permutation matrices.
- Parameters: the time duration for one lap of the raceway pond T and the light attenuation ratio q.

Numerical results

When $T = 1000\,\mathrm{s}$ and $q = 1\%$

$$P_{\max} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Numerical results

When $T = 1000\,\mathrm{s}$ and $q = 0.1\%$

$$P_{\max} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$P_{\max} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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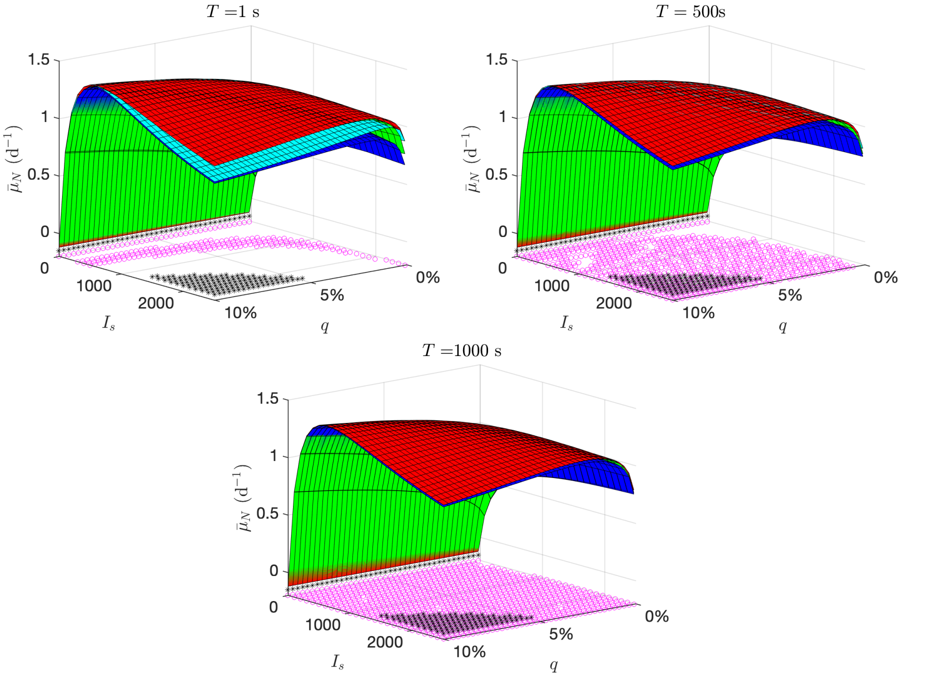

T =1 s
T = 500s
T =1000 s
1.5
1
0.5
0
$\bar{\mu}_N$ (d^{-1})
1000
2000
I_s
0%
5%
10%
q

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