Approches Lagrangiennes pour la modélisation et l'optimisation du couplage hydrodynamique-photosynthèse

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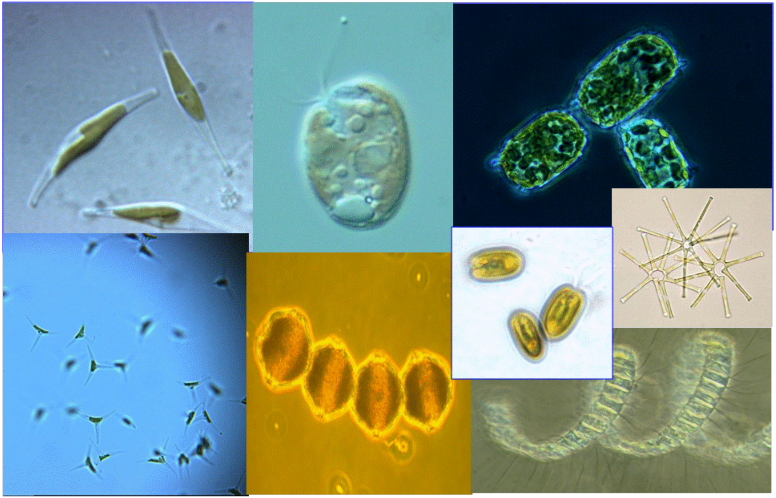
Wednesday, September 29, 2021

Overview

Motivation and Framework

- Microalgae:
  - photosynthetic micro-organisms,
  - 2 to 50 micro-meters,



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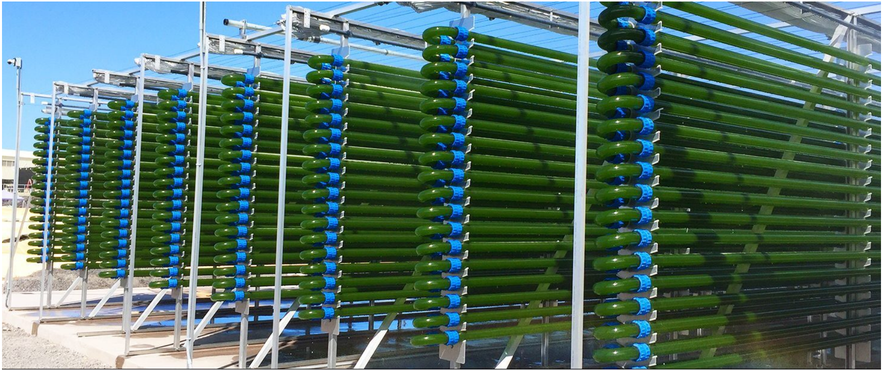

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open

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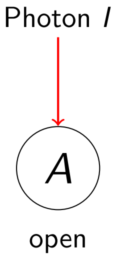

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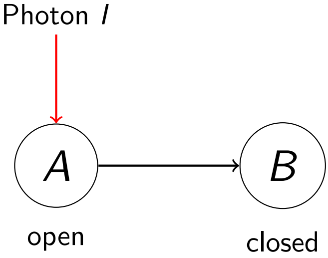

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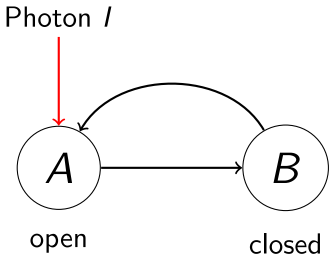

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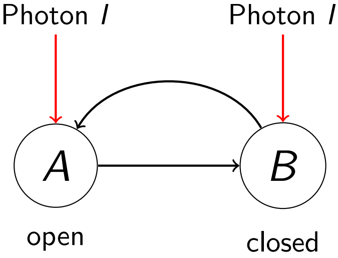

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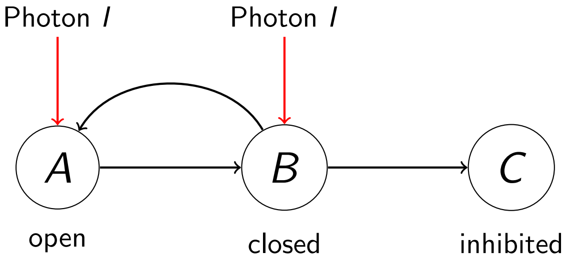

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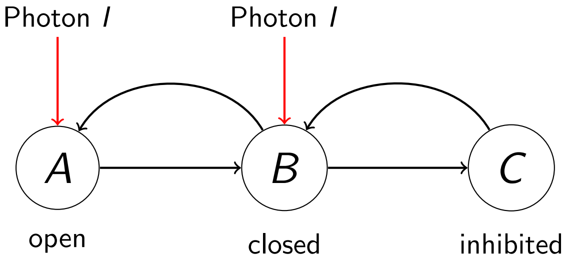

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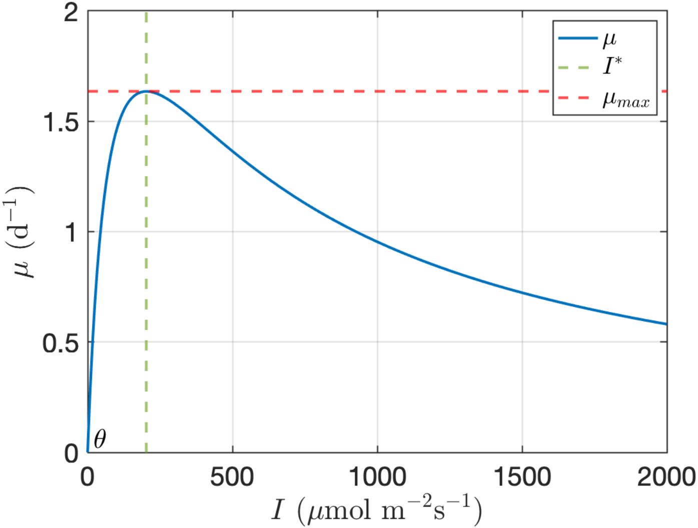
Haldane description



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- The Beer-Lambert law: $I(z) = I_s \exp\Big(-\varepsilon z \Big)$.

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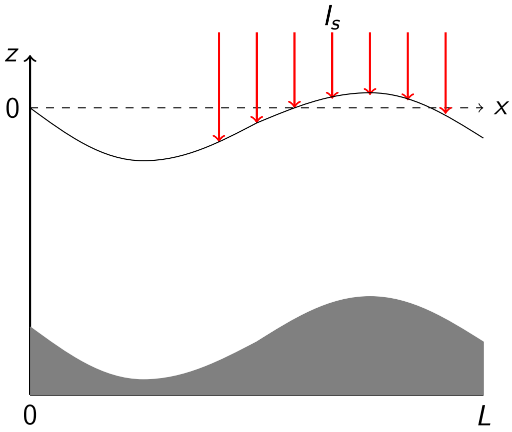

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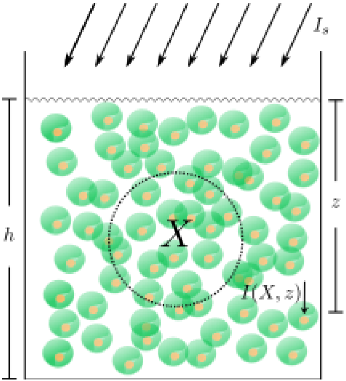
Masci et al. 2010:

- Growth μ: Droop function.



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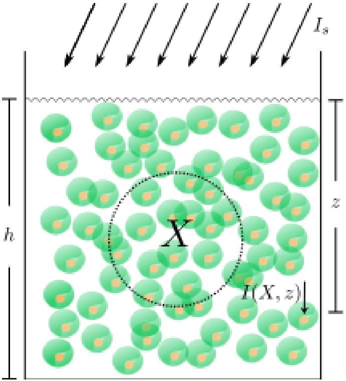

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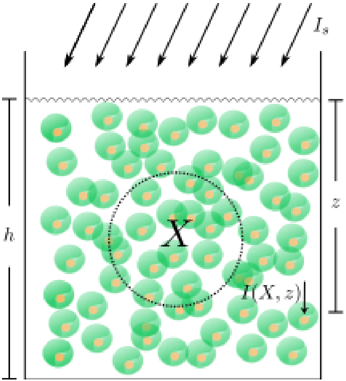

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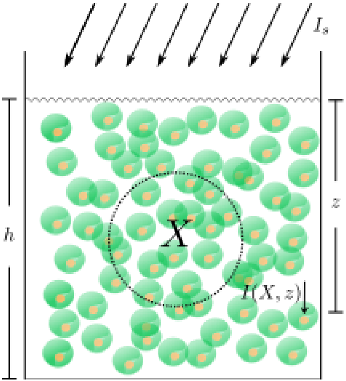

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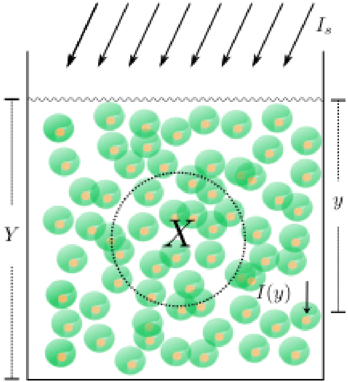

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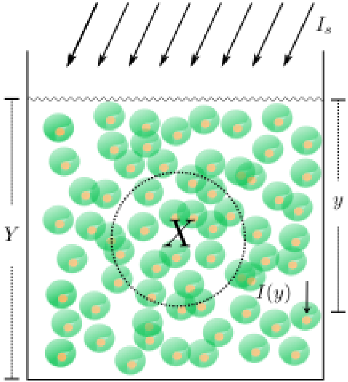

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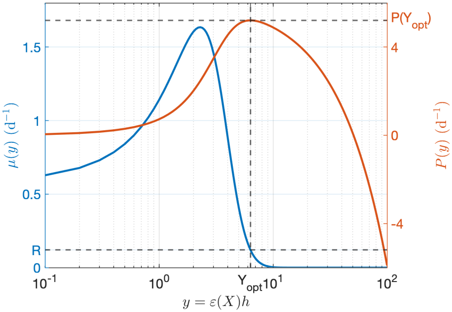

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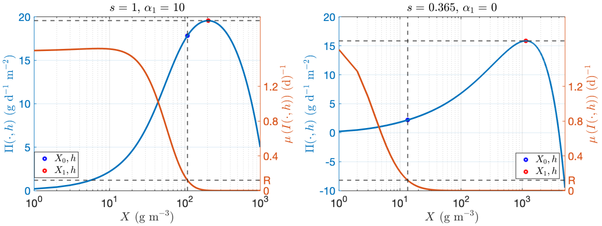
- For a given depth h, Y_{opt} is generally NOT the optimal condition.



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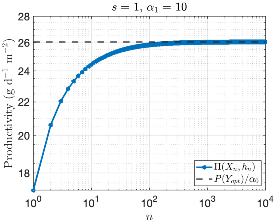
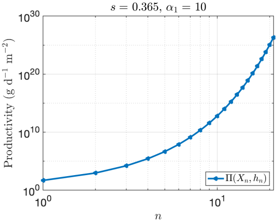
If $s = 1$, $\lim_{n\to\infty}\Pi(X_n, h_n) = \frac{P(Y_{opt})}{\alpha_0}$. If $s < 1$, $\lim_{n\to\infty}\Pi(X_n, h_n) = +\infty$.





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Proposition

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$$D = \begin{cases} D_{\max} & X \geq \bar{X} \\ (\bar{\mu}(X,h) - R)\frac{X}{X^\star} & X < \bar{X} \end{cases}$$

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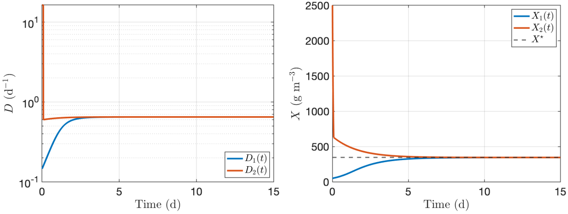

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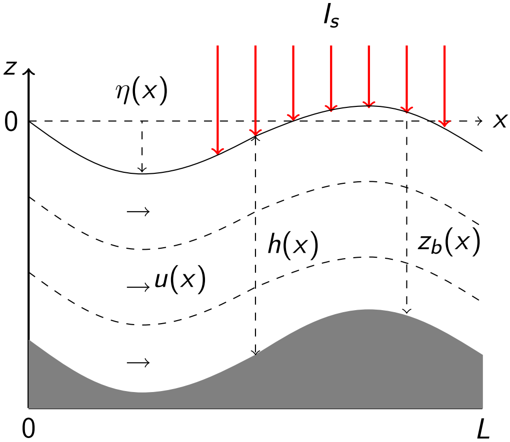

Saint-Venant Equations

- 1D steady state Saint-Venant equations

$$\partial_x(hu) = 0, \quad \partial_x(hu^2 + g\frac{h^2}{2}) = -gh\partial_x z_b.$$

Saint-Venant Equations

- Relation between z_b and h

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- Given a smooth topography z_b, there exists **a unique** positive smooth solution of h which satisfies the subcritical flow condition (*Michel-Dansac et al* 2016).
- A **time free** formulation of the Lagrangian trajectory starting from $z(0)$:
$$z(x) = \eta(x) + \frac{h(x)}{h(0)}(z(0) - \eta(0)). \tag{2}$$

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- Parameterize h by a vector $a := [a_1, \cdots, a_{N_a}] \in \mathbb{R}^{N_a}$, e.g. Truncated Fourier.

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- Objective function: Average net growth rate

$$\bar{\mu}_\infty := \frac{1}{V}\int_0^L \int_{z_b(x)}^{\eta(x)} \mu\big(C(x,z), I(x,z)\big)\mathrm{d}z\mathrm{d}x$$

$$\Big\downarrow \text{ vertical discretization}$$

$$\bar{\mu}_{N_z} = \frac{1}{VN_z}\sum_{i=1}^{N_z}\int_0^L \mu(C_i, I_i)h\mathrm{d}x$$

- Volume of the system $V = \int_0^L h(x)\mathrm{d}x$.
- Parameterize h by a vector $a := [a_1, \cdots, a_{N_a}] \in \mathbb{R}^{N_a}$, e.g. Truncated Fourier.
- The computational chain:

$$h(a) \to z_i \to I_i \to C_i \to \bar{\mu}_{N_z}.$$

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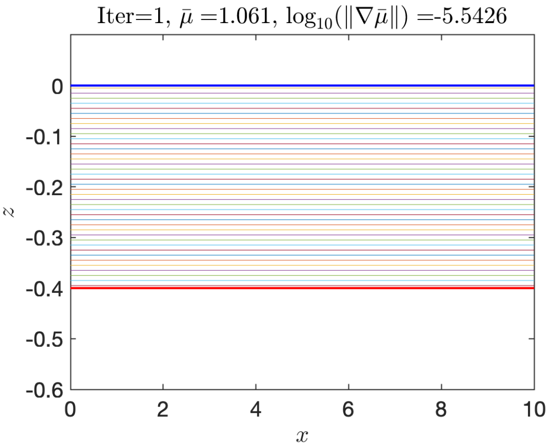
- The computational chain:

$$h(a) \to z_i \to I_i \to C_i \to \bar{\mu}_{N_z}.$$

- Adjoint method $\to$ $\nabla \bar{\mu}_{N_z}(a)$.

Optimal Topography

- Number of parameters: $N_a = 5$.
- Number of trajectories: $N_z = 40$.
- Initial guess: flat topography.



Permanent regime

Assumption

Photoinhibition state C is periodic meaning that $C_i(L) = C_i(0)$, $i = [1, \cdots, N_z]$.

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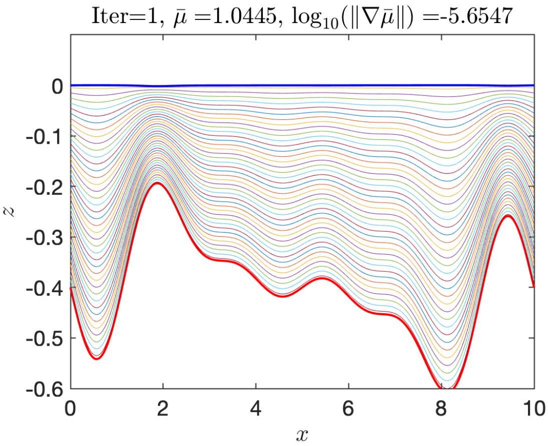
Photoinhibition state C is periodic meaning that $C_i(L) = C_i(0)$, $i = [1, \cdots, N_z]$.

Theorem (Flat topography)

Assume the volume of the system V is constant. Then $\nabla \bar{\mu}_{N_z}(0) = 0$.

Optimal topography (C periodic)

- Number of parameters: $N_a = 5$.
- Number of trajectories: $N_z = 40$.
- Initial guess: random topography.



Summary on the topography

- In the case C non periodic, one can find no flat optimal topographies, however the increase is limited.

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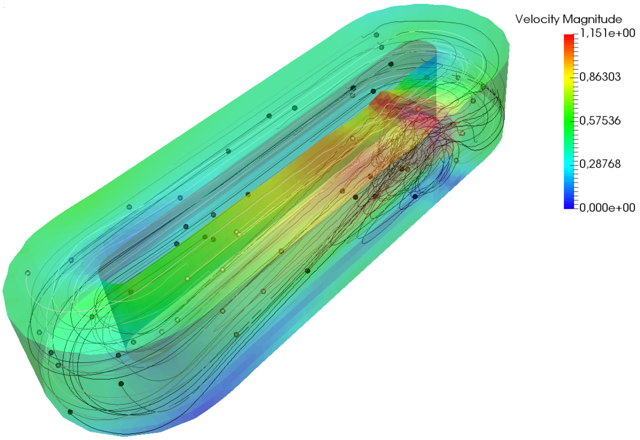
- In the case C non periodic, one can find no flat optimal topographies, however the increase is limited.
- In the case C periodic, the flat topography is not only a critical point but also the optimal topography.
- What can be further optimized?

Overview

Mixing devices

Simulation of the trajectories with the code FreshKiss3D (*Demory et al.* 2018).



Mixing devices

Assumption (Ideal rearrangement)

At each new lap, the algae at depth z_i **are entirely transferred into the position** z_j when passing through the mixing device.

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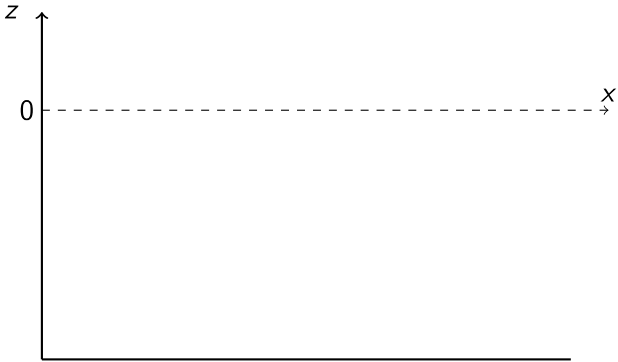
Notations

We denote by $\mathcal{P}$ the set of **permutation matrices** of size $N_z \times N_z$ and by $\mathfrak{S}_{N_z}$ the associated set of permutations of N_z elements.

Mixing devices

- Illustration with the permutation $\sigma = (1\ 2\ 3\ 4)$.



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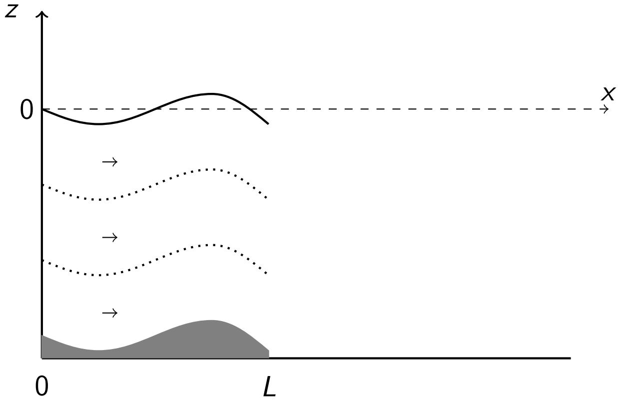

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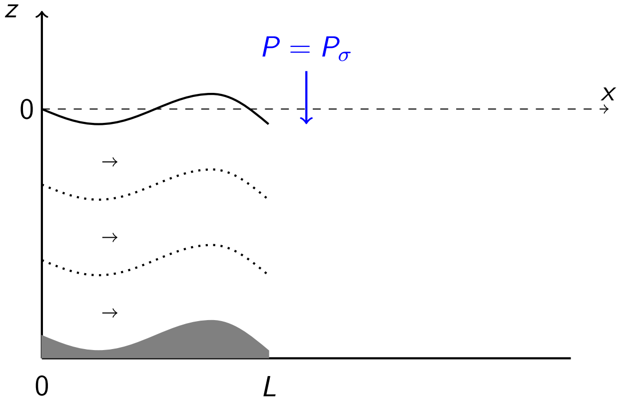

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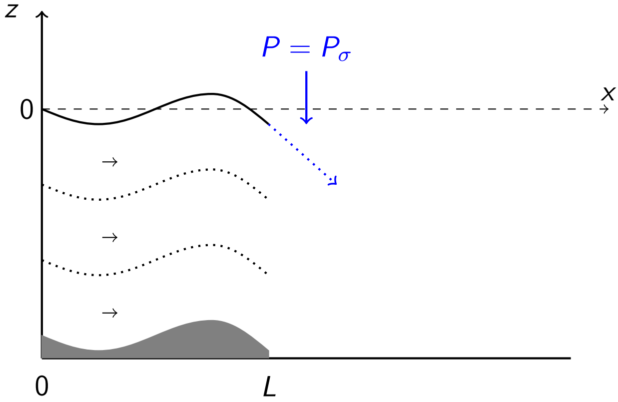

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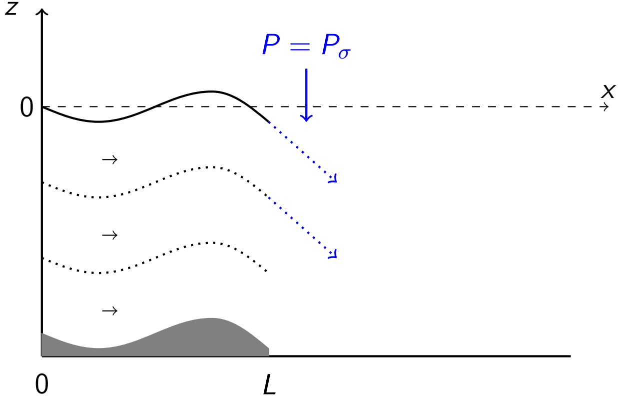

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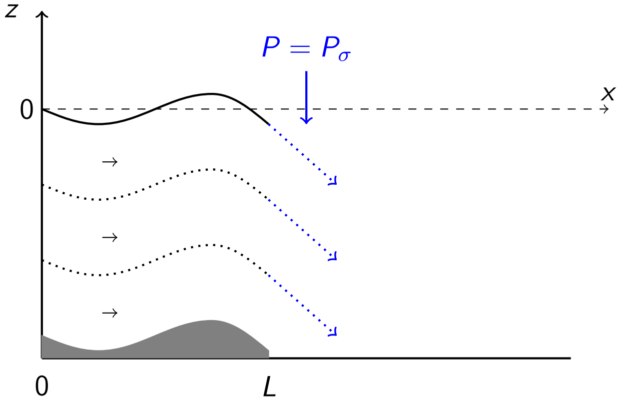

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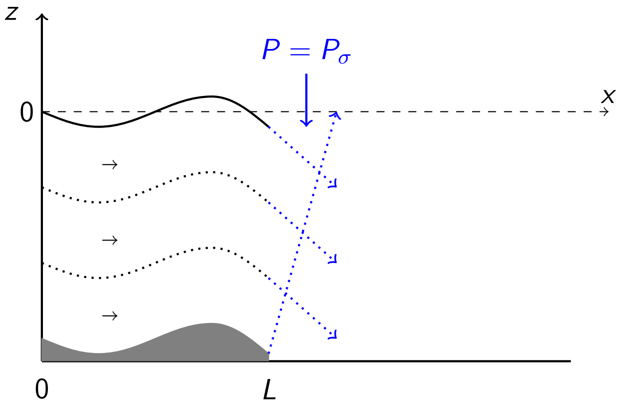

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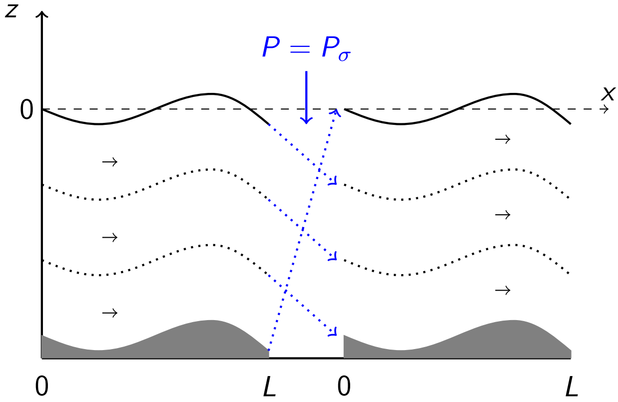

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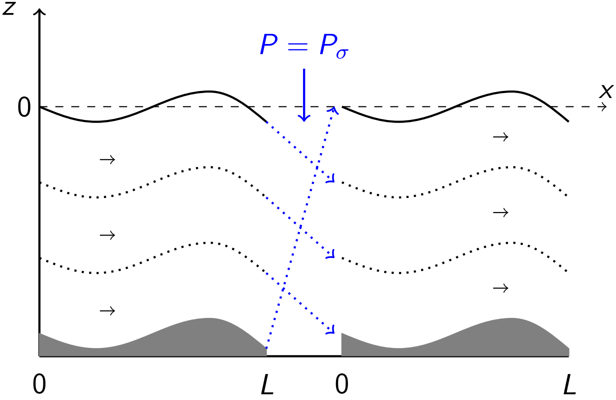

- Choice of Period?

Mixing devices

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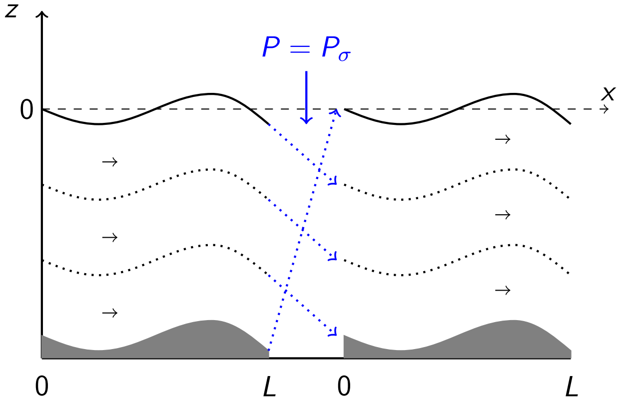

- Choice of Period? Order of σ.

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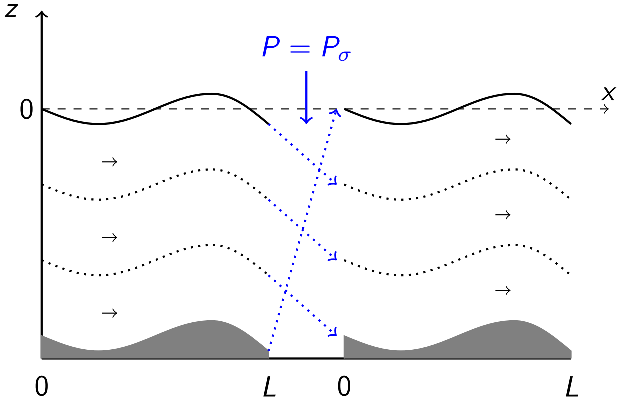
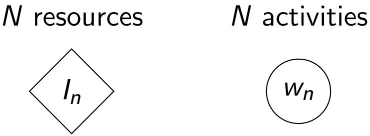


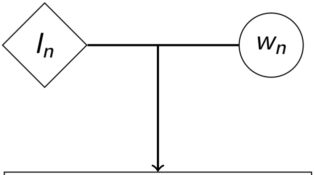
- Choice of Period? Order of σ.
- Re-distribution of light.

Periodic dynamical resource allocation problem



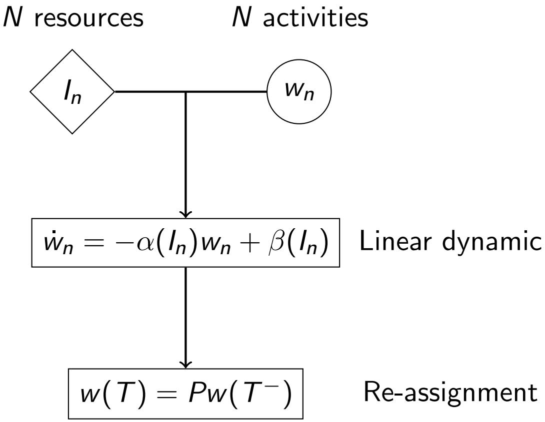
Periodic dynamical resource allocation problem

N resources $\qquad$ N activities



$$\boxed{\dot{w}_n = -\alpha(I_n) w_n + \beta(I_n)}$$ Linear dynamic

Periodic dynamical resource allocation problem



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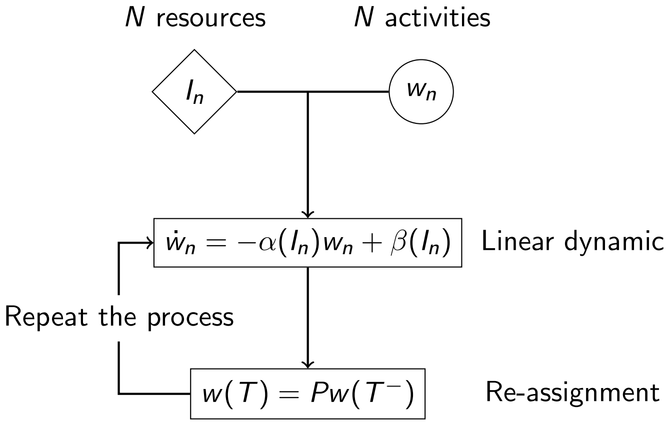

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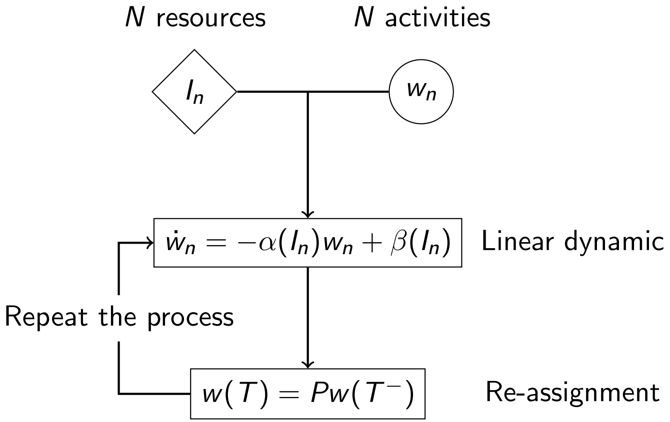

Theorem (One period is enough)

If w is KT-periodic (i.e., $w(T_K) = w(T_0)$), then w is $T-$periodic.

Original problem

Optimization problem

$$\max_{P\in\mathcal{P}} J(P) := \max_{P\in\mathcal{P}} \langle u, (\mathcal{I}_N - PD)^{-1} Pv \rangle, \tag{3}$$

Two vectors u, v and a diagonal matrix D all depend on $(l_n)_{n=1}^N$.

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Remark

Since $\#\mathfrak{S} = N!$, this problem cannot be tackled in realistic cases where large values of N must be considered, e.g., to keep a good numerical accuracy.

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Since $\#\mathfrak{S} = N!$, this problem cannot be tackled in realistic cases where large values of N must be considered, e.g., to keep a good numerical accuracy.

Expand the functional (3) as follows

$$\underbrace{\langle u, (\mathcal{I}_N - PD)^{-1} Pv \rangle}_{J(P)} = \sum_{\ell=0}^{+\infty} \langle u, (PD)^{\ell} Pv \rangle = \underbrace{\langle u, Pv \rangle}_{J^{\text{approx}}(P)} + \sum_{\ell=1}^{+\infty} \langle u, (PD)^{\ell} Pv \rangle,$$

Simplified problem

$$\max_{P\in\mathcal{P}} J^{\text{approx}}(P) := \max_{P\in\mathcal{P}} \langle u, Pv \rangle. \tag{4}$$

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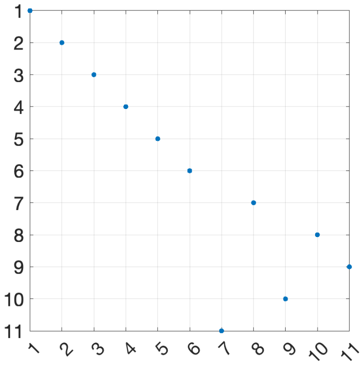
Lemma (Optimal matrix)

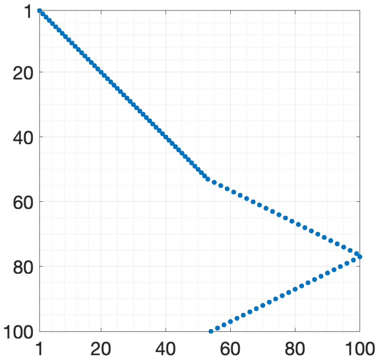
- P_+: *associates the largest coefficient of u with the largest coefficient of v, the second largest coefficient with the second largest, and so on.*
- P_-: *associates the largest coefficient of u with the smallest coefficient of v, the second largest coefficient with the second smallest, and so on.*

Optimal Matrix

Test for $(I_s, q, T) = (2000, 5\%, 1000)$.



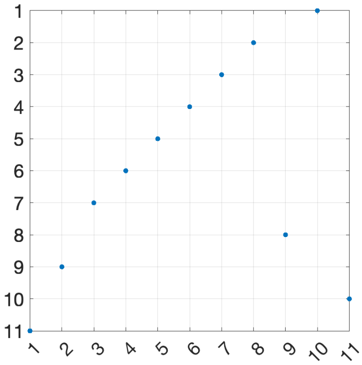
$P_{\max}$ for $J(P)$



P_{+} for $J^{\text{approx}}(P)$

Optimal Matrix

Test for $(I_s, q, T) = (800, 0.5\%, 1)$.



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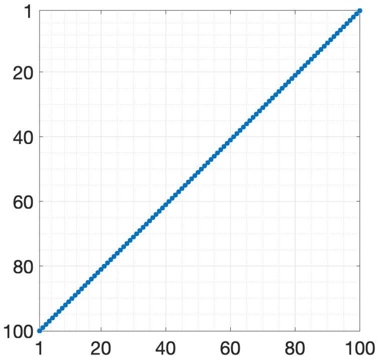


P_+ for $J^{\text{approx}}(P)$

Quality of the approximation

Theorem (Coincidence Criterion: $P_{\max} = P_+?$)

Assume that u and v have positive entries and define

$$\phi(m) := \frac{1}{s_{\lceil \frac{m}{2} \rceil}} \Big(\sum_{\ell=1}^{+\infty} d_{\max}^{\ell} F_{(\ell+1)m}^{+} - d_{\min}^{\ell} F_{(\ell+1)m}^{-} \Big), \tag{5}$$

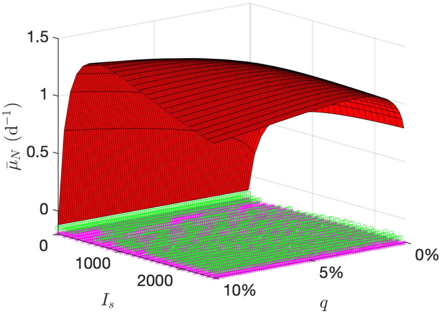
where $m := \# \{ n = 1, \ldots, N \mid \sigma(n) \neq \sigma_+(n) \}$, $d_{\max} := \max_{n=1,\ldots,N}(d_n)$ and $d_{\min} := \min_{n=1,\ldots,N}(d_n)$. Assume that:

$$\max_{m \geq 2} \phi(m) \leq 1. \tag{6}$$

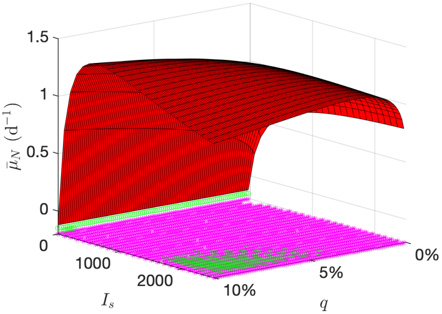
Then $P_{\max} = P_+$.

Approximation and criterion

$T = 1000$.



$N = 5$

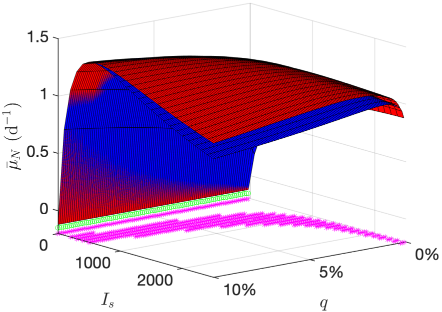


$N = 9$

- $\bar{\mu}_N(P_{\text{max}})$ and $\bar{\mu}_N(P_+)$.
- $P_{\text{max}} \quad = \quad P_+$.
- Coincidence Criterion satisfied.

Approximation and criterion

$T = 1.$



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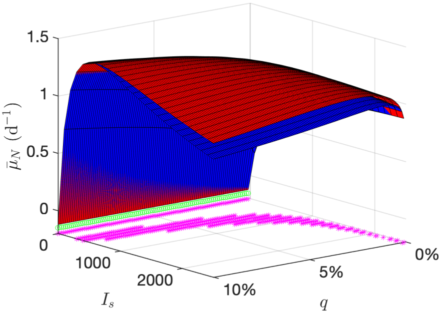


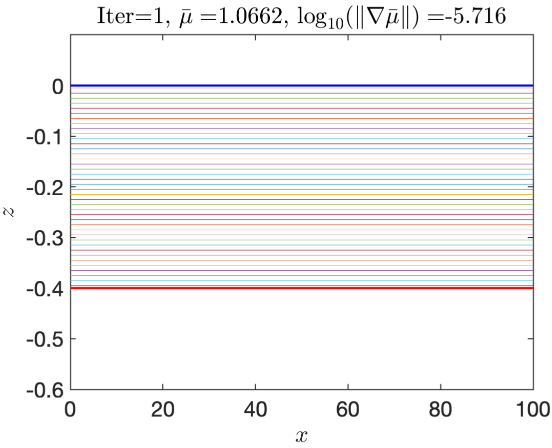
$N = 9$

- $\bar{\mu}_N(P_{\max})$ and $\bar{\mu}_N(P_+)$.
- $P_{\max} = P_+$.
- Coincidence Criterion satisfied

Overview

Test with a permutation

- Test permutation: $\sigma = (1\ N_z)(2\ N_z - 1)\ldots$.
- Initial guess: flat topography.



Variable volume

- Volume related parameter a_0 as the average depth of the raceway system:
$$a_0 := \bar{h} = \frac{1}{L} \int_0^L h(x) \mathrm{d}x = \frac{V}{L}. \tag{7}$$

New parameter $\tilde{a} = [a_0, a_1, \ldots, a_{N_a}] \in \mathbb{R}^{N_a+1}$.

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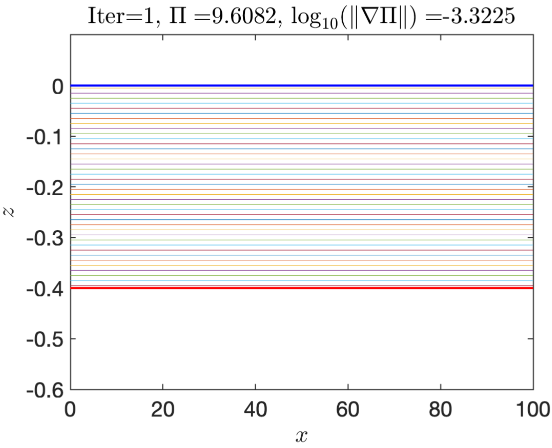
New parameter $\tilde{a} = [a_0, a_1, \ldots, a_{N_a}] \in \mathbb{R}^{N_a+1}$.
- Relation between X and V: Y_{opt}.
- Optimization Problem:

$$\Pi_{N_z}(\tilde{a}) := \bar{\mu}_{N_z}(\tilde{a})Xh(\tilde{a}) = \frac{Y_{\text{opt}} - \alpha_1 a_0}{VN_z\alpha_0}\sum_{i=1}^{N_z}\int_0^L \mu(C_i, I_i(\tilde{a}))h(\tilde{a})\mathrm{d}x.$$

Optimal Topography (Variable volume)

- Initial average depth: $a_0 = 0.4$m.
- Initial guess: flat topography.



Optimal Topography (Variable volume)

- Number of trajectories: $N_z = 7$.
- Initial average depth: $a_0 = 0.4$m.
- Initial guess: flat topography.

$$P_{\max}^{100} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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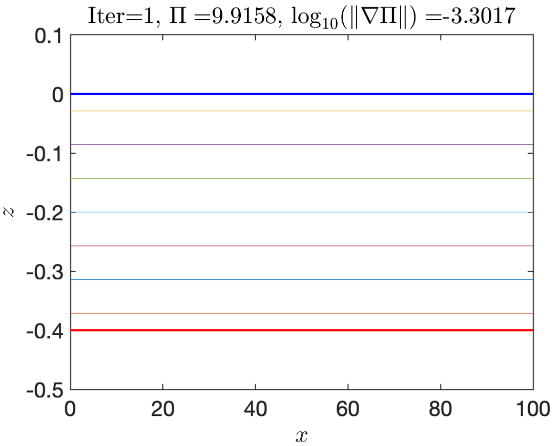

Overview

Conclusion

Depth / Biomass concentration:

- Optical depth productivity.
- Optimal condition to maximize the productivity.
- Nonlinear controller to stabilize the biomass concentration.

Topography:

- Flat topography is optimal in periodic case.
- Non flat topography with limited increase.

Mixing:

- Periodic dynamic resource allocation problem.
- One period is enough.
- Approximation and criterion.

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	Topography	Mixing	Depth / Biomass concentration
Gain	≈ 1 %	≈ 30 %	≈ 100 %

Contribution

Submitted paper:

- O. Bernard, L.-D. Lu, J. Sainte-Marie and J. Salomon, Shape optimization of a microalgal raceway to enhance productivity. *Submitted paper*, November 2020.
- O. Bernard, L.-D. Lu and J. Salomon, Optimization of mixing strategy in microalgal raceway ponds. *Submitted paper*, March 2021.
- O. Bernard and L.-D. Lu, Optimal optical conditions for Microalgal production in photobioreactors. *Submitted paper*, August 2021.

Conference proceeding:

- O. Bernard, L.-D. Lu, J. Sainte-Marie and J. Salomon, Controlling the bottom topography of a microalgal pond to optimize productivity. *2021 American Control Conference (ACC)* , pages 634–639, 2021.
- O. Bernard, L.-D. Lu and J. Salomon, Optimizing microalgal productivity in raceway ponds through a controlled mixing device. *2021 American Control Conference (ACC)* , pages 640–645, 2021.
- O. Bernard, L.-D. Lu and J. Salomon, Mixing strategies combined with shape design to enhance productivity of a raceway pond. *16th IFAC Symposium on Advanced Control of Chemical Processes ADCHEM 2021*, pages 281-286, 2021.

Future work

New approach where topography, mixing, depth and biomass concentration have been combined.
To be further investigated:

- Improve the criterion.
- Provide an approximation for small time period.
- Theoretical proof of the optimal flat topography.
- Test influence of the topography with other models.

Future work

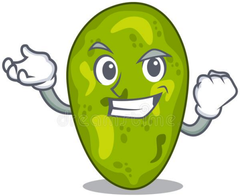
Further step that can lead to higher gains:

- Consider the turbulence regime (much more complex...).

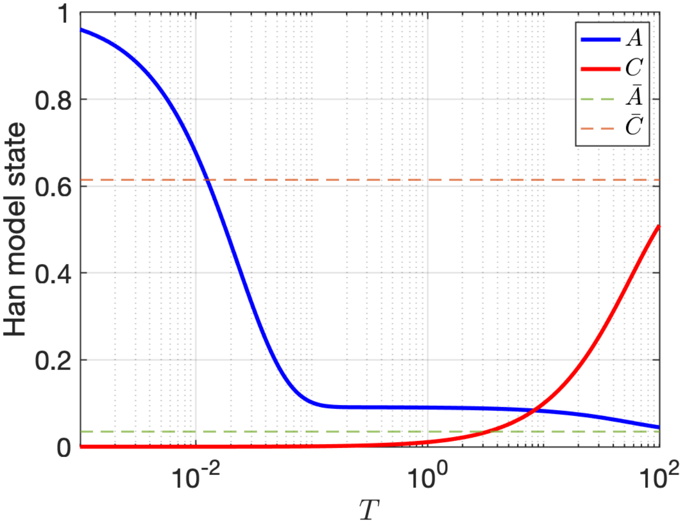
But for this:

- Include the faster time scales of the Han model.
- A more refined model of the mixing device (and its implication on hydrodynamics) must be developed.
- Higher energetic cost for maintaining a turbulent regime must be taken into account.

Thanks for your attention

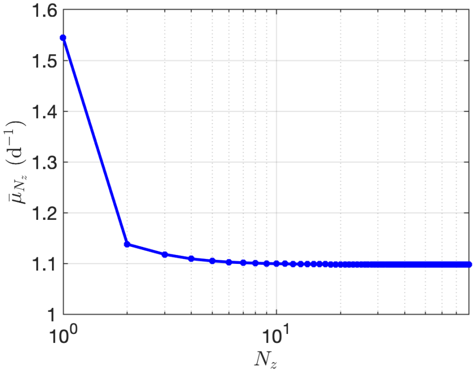


Fast/slow illustration



Effect on vertical discretization number

We fix $N_a = 5$ and take 100 random vector a. For N_z varying from 1 to 80, we compute the average value of $\bar{\mu}_{N_z}$ for each N_z.



Objective function

Define the average benefit after K operations

$$\frac{1}{K}\sum_{k=0}^{K-1}\langle u, \frac{1}{T}\int_{T_k}^{T_{k+1}} x(t)\mathrm{d}t\rangle.$$

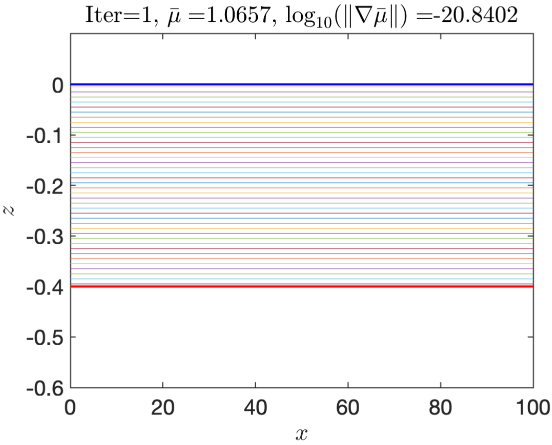
Theorem (One periodic)

If x is KT-periodic (i.e., $x(T_K) = x(T_0)$), then x is $T-$periodic.

$$\frac{1}{K}\sum_{k=0}^{K-1}\langle u, \frac{1}{T}\int_{T_k}^{T_{k+1}} x(t)\mathrm{d}t\rangle = \langle u, \frac{1}{T}\int_{T_0}^{T_1} x(t)\mathrm{d}t\rangle.$$

Test with a permutation

We keep $N_a = 5$, $N_z = 40$ and choose $\sigma = Id$



One periodic

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