Microalgal raceway ponds modeling and optimization problems

Liu-Di LU

Tuesday, November 2, 2021

Overview

Motivation and Framework

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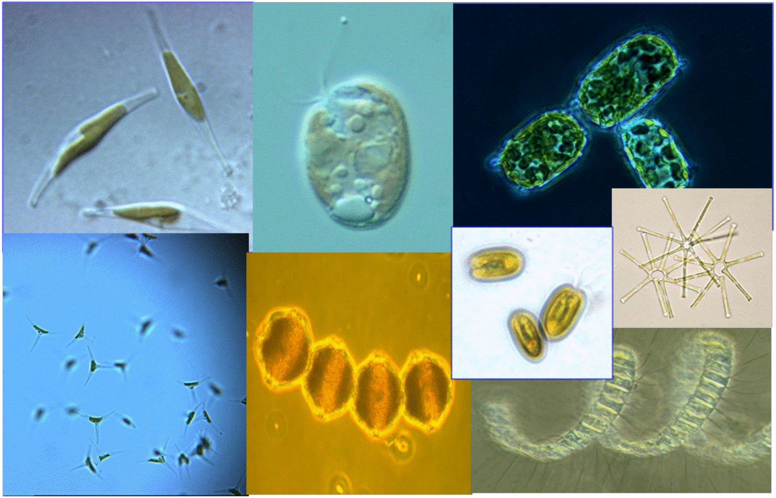
  - photosynthetic micro-organisms,
  - 2 to 50 micro-meters,



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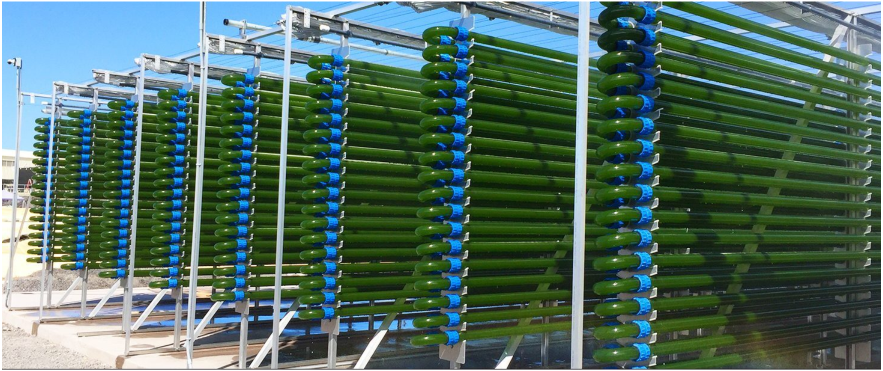

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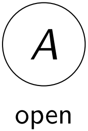

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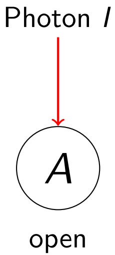

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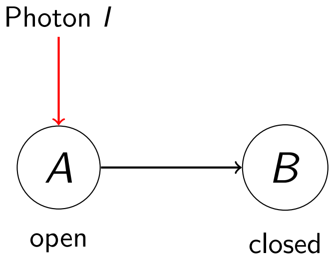

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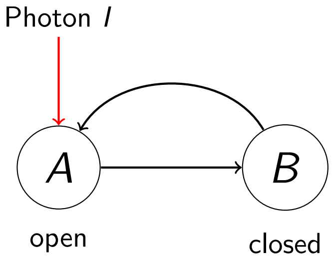

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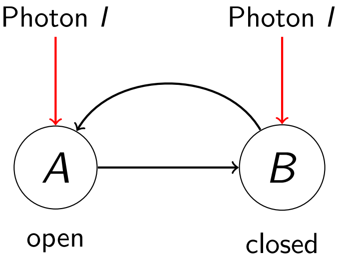

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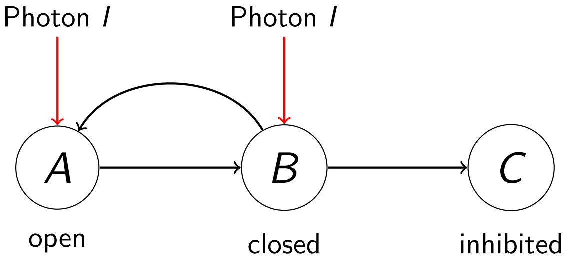

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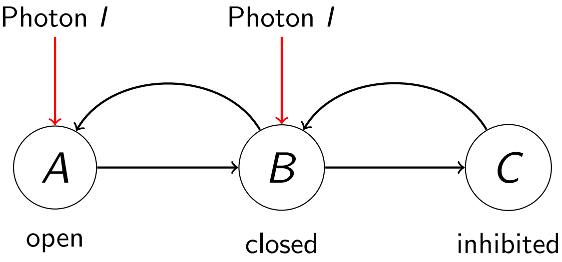

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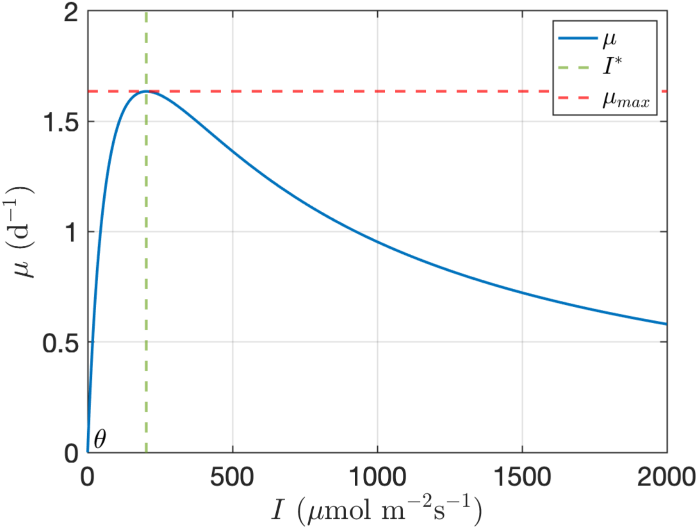
Haldane description



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- The Beer-Lambert law: $I(z) = I_s \exp\Big(- \varepsilon z \Big)$.

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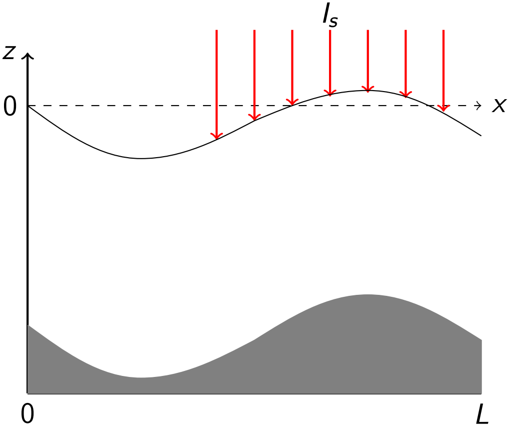

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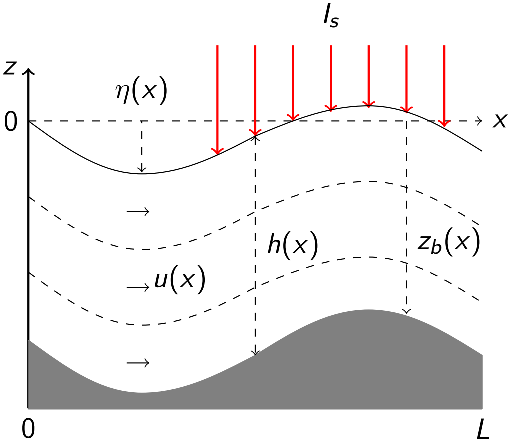

Saint-Venant Equations

- 1D steady state Saint-Venant equations

$$\partial_x(hu) = 0, \quad \partial_x(hu^2 + g\frac{h^2}{2}) = -gh\partial_x z_b.$$

Saint-Venant Equations

- Relation between z_b and h

$$z_b = \frac{M_0}{g} - \frac{Q_0^2}{2gh^2} - h, \tag{1}$$

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- Given a smooth topography z_b, there exists **a unique** positive smooth solution of h which satisfies the subcritical flow condition (*Michel-Dansac et al* 2016).
- A **time free** formulation of the Lagrangian trajectory starting from $z(0)$:

$$z(x) = \eta(x) + \frac{h(x)}{h(0)}(z(0) - \eta(0)). \tag{2}$$

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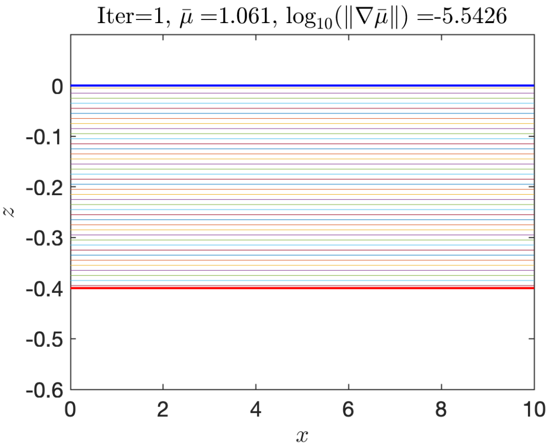
- Adjoint method $\to$ $\nabla\bar{\mu}_{N_z}(a)$.

Optimal Topography

- Number of parameters: $N_a = 5$.
- Number of trajectories: $N_z = 40$.
- Initial guess: flat topography.



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Theorem (Flat topography)

Assume the volume of the system V is constant. Then $\nabla \bar{\mu}_{N_z}(0) = 0$.

Optimal topography (C periodic)

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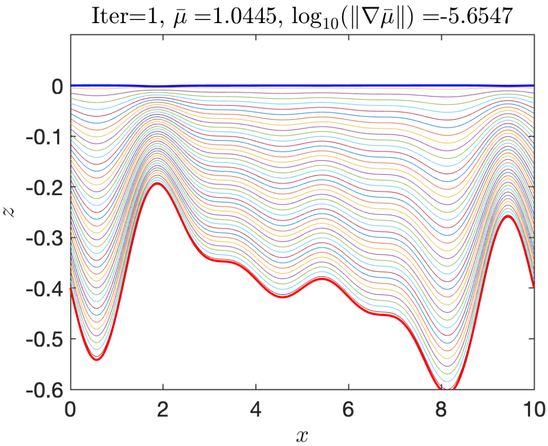

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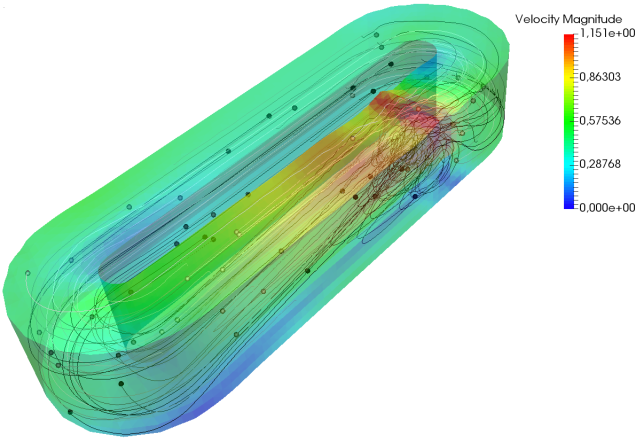
- In the case C periodic, the flat topography is not only a critical point but also the optimal topography.
- What can be further optimized?

Overview

Mixing devices

Simulation of the trajectories with the code FreshKiss3D (*Demory et al.* 2018).



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Notations

We denote by $\mathcal{P}$ the set of **permutation matrices** of size $N_z \times N_z$ and by $\mathfrak{S}_{N_z}$ the associated set of permutations of N_z elements.

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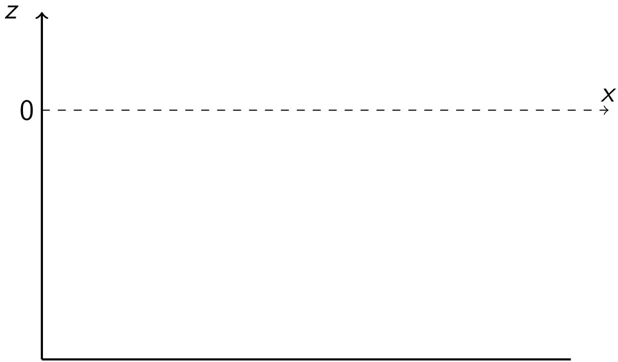
- Illustration with the permutation $\sigma = (1\ 2\ 3\ 4)$.



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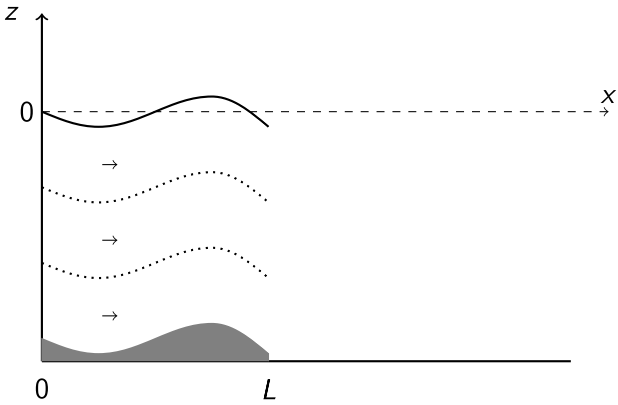

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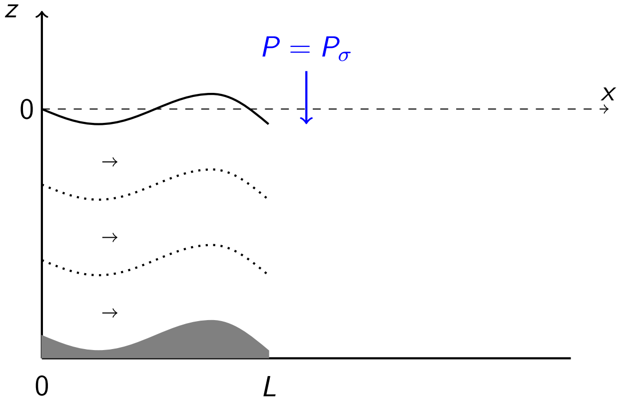

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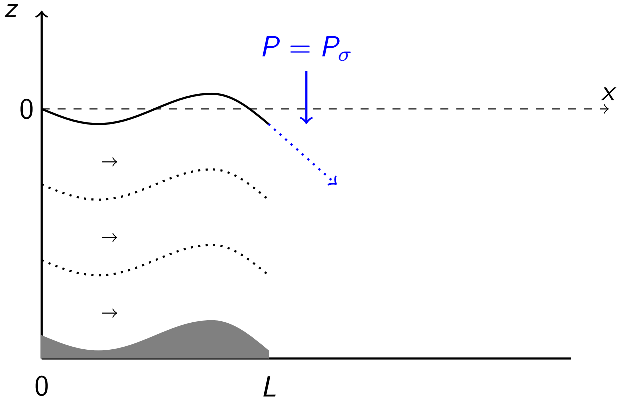

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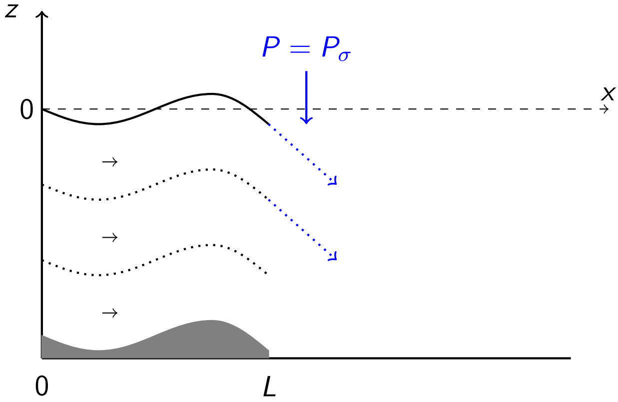

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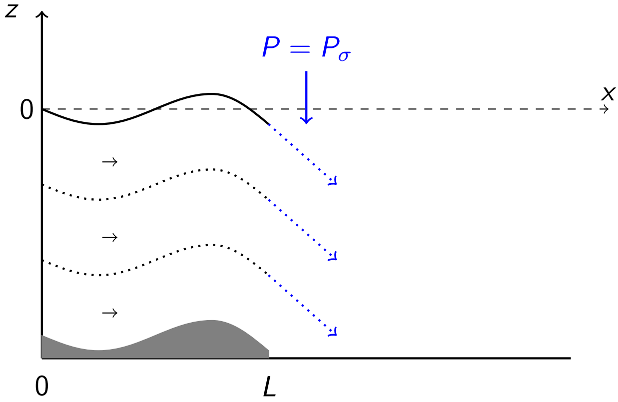

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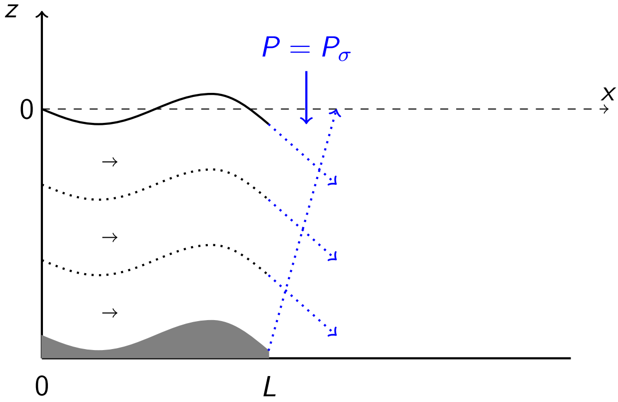

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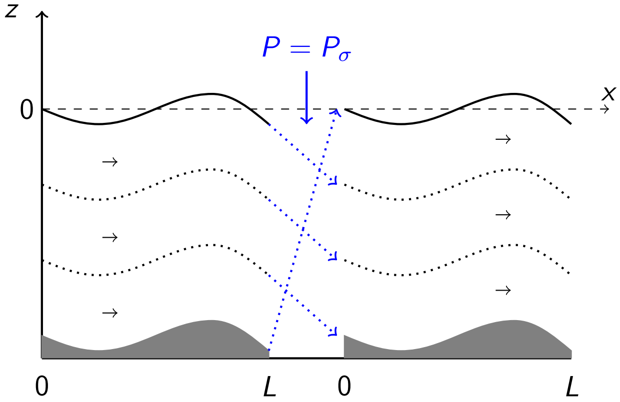

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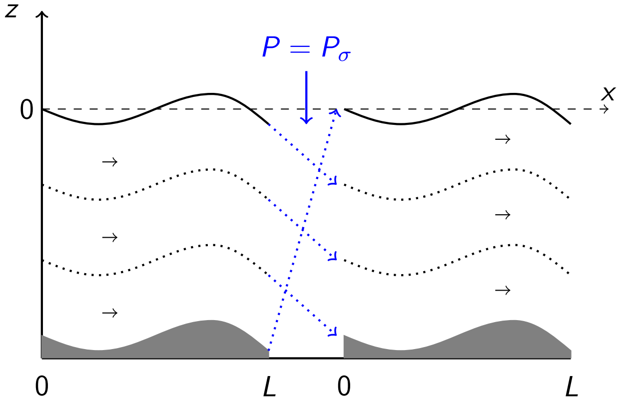

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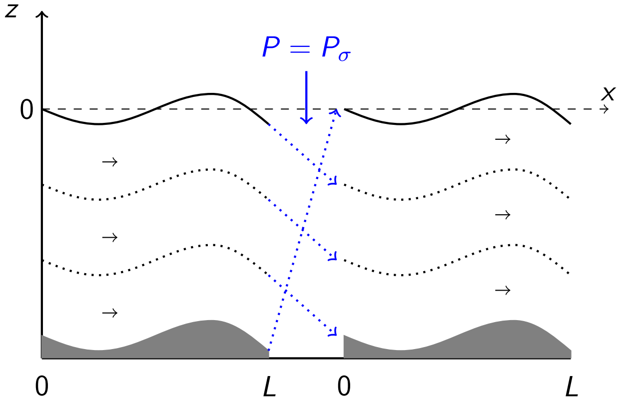

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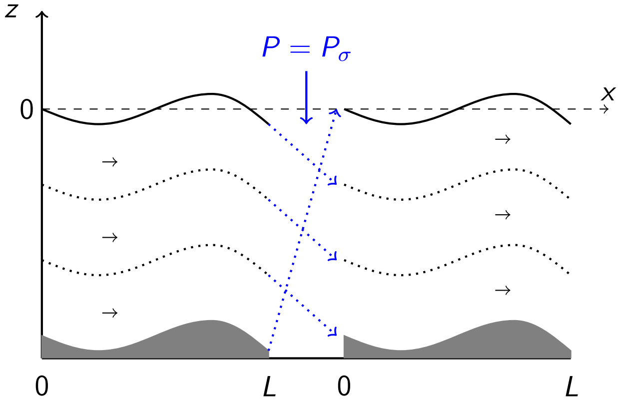

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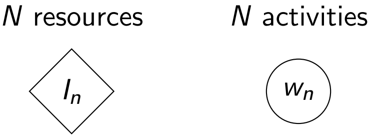
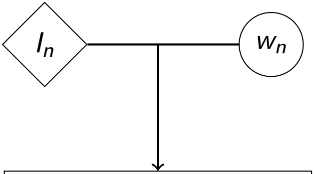
- Re-distribution of light.

Periodic dynamical resource allocation problem



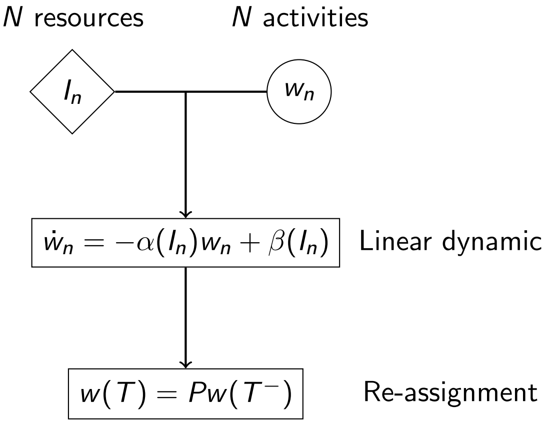
Periodic dynamical resource allocation problem

N resources $\qquad$ N activities

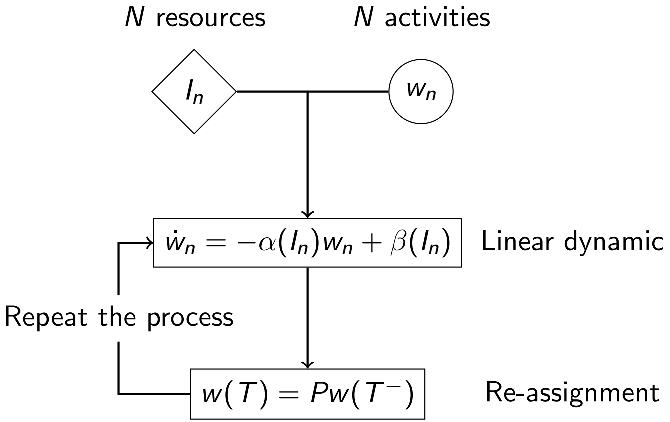


$$\boxed{\dot{w}_n = -\alpha(I_n)w_n + \beta(I_n)}$$ Linear dynamic

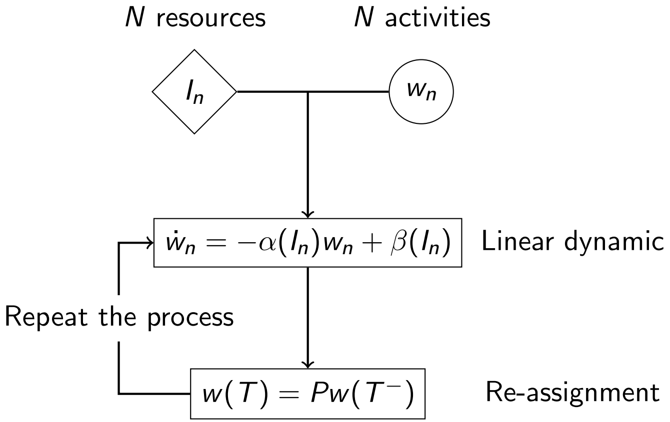
Periodic dynamical resource allocation problem



Periodic dynamical resource allocation problem



Periodic dynamical resource allocation problem



Theorem (One period is enough)

If w is KT-periodic (i.e., $w(T_K) = w(T_0)$), then w is $T-$periodic.

Original problem

Optimization problem

$$\max_{P\in\mathcal{P}} J(P) := \max_{P\in\mathcal{P}} \langle u, (\mathcal{I}_N - PD)^{-1} Pv \rangle, \tag{3}$$

Two vectors u, v and a diagonal matrix D all depend on $(I_n)_{n=1}^{N}$.

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Remark

Since $\#\mathfrak{S} = N!$, this problem cannot be tackled in realistic cases where large values of N must be considered, e.g., to keep a good numerical accuracy.

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Expand the functional (3) as follows

$$\underbrace{\langle u, (\mathcal{I}_N - PD)^{-1} Pv\rangle}_{J(P)} = \sum_{\ell=0}^{+\infty} \langle u, (PD)^{\ell} Pv\rangle = \underbrace{\langle u, Pv\rangle}_{J^{\text{approx}}(P)} + \sum_{\ell=1}^{+\infty} \langle u, (PD)^{\ell} Pv\rangle,$$

Simplified problem

$$\max_{P\in\mathcal{P}} J^{\text{approx}}(P) := \max_{P\in\mathcal{P}} \langle u, Pv\rangle. \tag{4}$$

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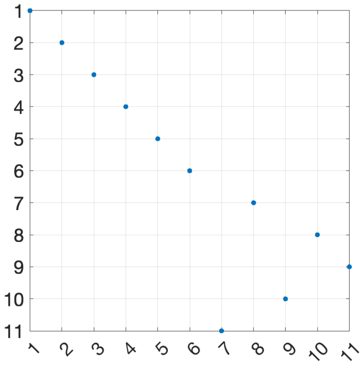
Lemma (Optimal matrix)

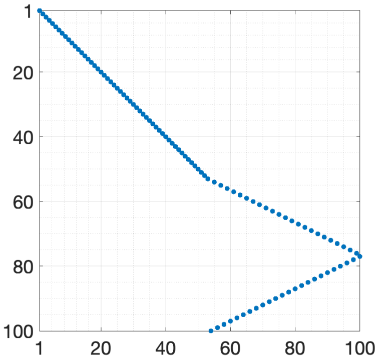
- *P_+: associates the largest coefficient of u with the largest coefficient of v, the second largest coefficient with the second largest, and so on.*
- *P_-: associates the largest coefficient of u with the smallest coefficient of v, the second largest coefficient with the second smallest, and so on.*

Optimal Matrix

Test for $(I_s, q, T) = (2000, 5\%, 1000)$.



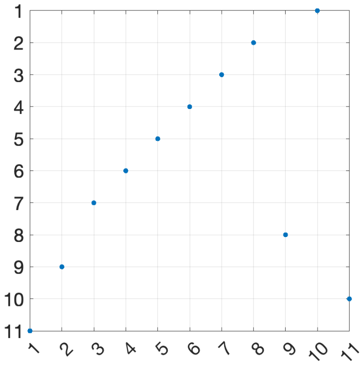
$P_{\max}$ for $J(P)$



P_+ for $J^{\text{approx}}(P)$

Optimal Matrix

Test for $(I_s, q, T) = (800, 0.5\%, 1)$.



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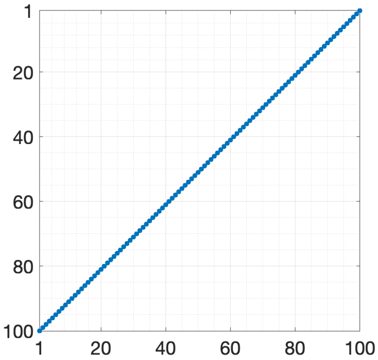


P_+ for $J^{\mathrm{approx}}(P)$

Quality of the approximation

Theorem (Coincidence Criterion: $P_{\max} = P_+$?)

Assume that u and v have positive entries and define

$$\phi(m) := \frac{1}{s_{\lceil \frac{m}{2} \rceil}} \Big(\sum_{\ell=1}^{+\infty} d_{\max}^{\ell} F^{+}_{(\ell+1)m} - d_{\min}^{\ell} F^{-}_{(\ell+1)m} \Big), \tag{5}$$

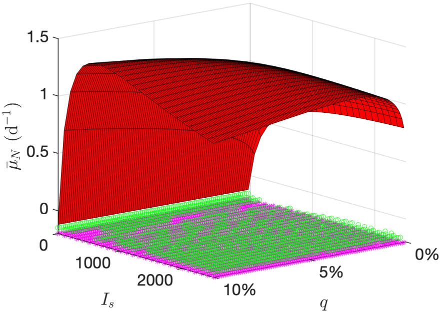
where $m := \#\{n = 1, \ldots, N \mid \sigma(n) \neq \sigma_+(n)\}$, $d_{\max} := \max_{n=1,\ldots,N}(d_n)$ and $d_{\min} := \min_{n=1,\ldots,N}(d_n)$. Assume that:

$$\max_{m \geq 2} \phi(m) \leq 1. \tag{6}$$

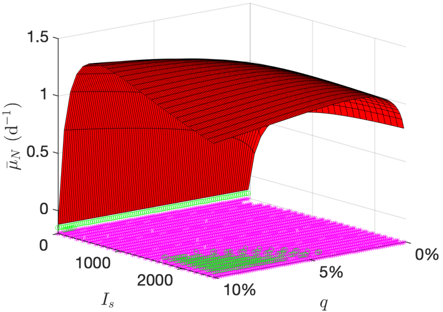
Then $P_{\max} = P_+$.

Approximation and criterion

$T = 1000$.



$N = 5$

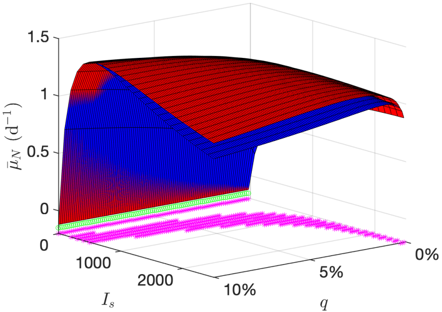


$N = 9$

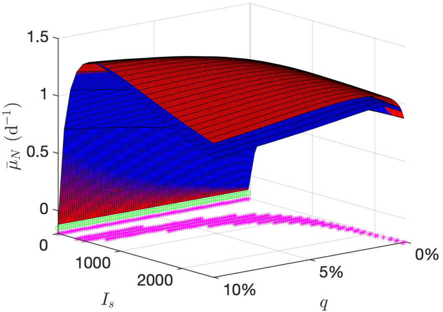
- $\bar{\mu}_N(P_{\max})$ and $\bar{\mu}_N(P_+)$.
- $P_{\max} \quad = \quad P_+$.
- Coincidence Criterion satisfied.

Approximation and criterion

$T = 1$.



$N = 5$



$N = 9$

- $\bar{\mu}_N(P_{\max})$ and $\bar{\mu}_N(P_+)$.
- $P_{\max} \quad = \quad P_+$.
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Overview

Conclusion

Topography:

- Flat topography is optimal in periodic case.
- Non flat topography with limited increase.

Mixing:

- Periodic dynamic resource allocation problem.
- One period is enough.
- Approximation and criterion.

Conclusion

Topography:

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- Non flat topography with limited increase.

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	Topography	Mixing
Gain	≈ 1 %	≈ 30 %

Future work

Further step that can lead to higher gains:

- Consider the turbulence regime (much more complex...).

But for this:

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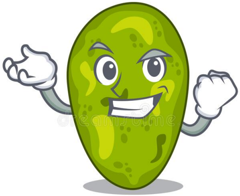
Further step that can lead to higher gains:

- Consider the turbulence regime (much more complex...).

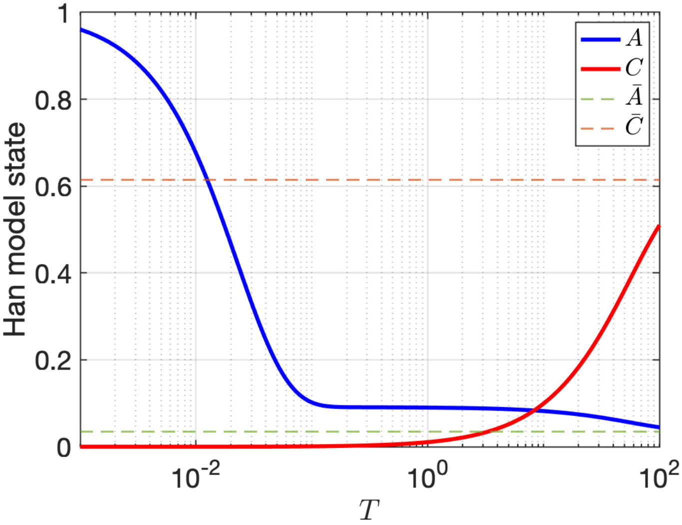
But for this:

- Include the faster time scales of the Han model.
- A more refined model of the mixing device (and its implication on hydrodynamics) must be developed.
- Higher energetic cost for maintaining a turbulent regime must be taken into account.

Thanks for your attention

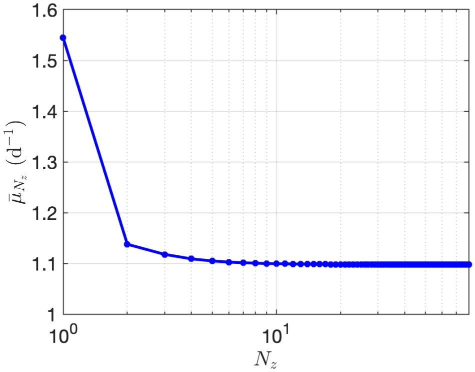


Fast/slow illustration



Effect on vertical discretization number

We fix $N_a = 5$ and take 100 random vector a. For N_z varying from 1 to 80, we compute the average value of $\bar{\mu}_{N_z}$ for each N_z.



Objective function

Define the average benefit after K operations

$$\frac{1}{K}\sum_{k=0}^{K-1}\langle u, \frac{1}{T}\int_{T_k}^{T_{k+1}} x(t)\mathrm{d}t\rangle.$$

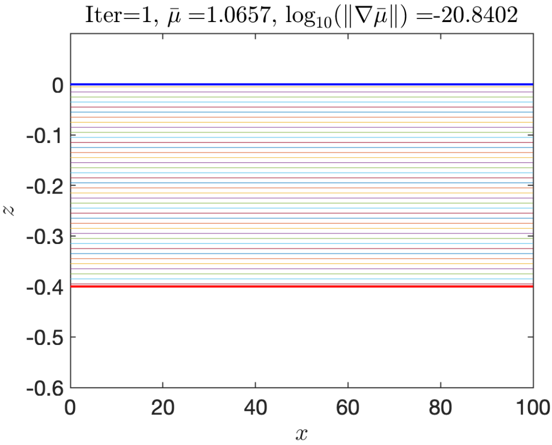
Theorem (One periodic)

If x is KT-periodic (i.e., $x(T_K) = x(T_0)$), then x is $T-$periodic.

$$\frac{1}{K}\sum_{k=0}^{K-1}\langle u, \frac{1}{T}\int_{T_k}^{T_{k+1}} x(t)\mathrm{d}t\rangle = \langle u, \frac{1}{T}\int_{T_0}^{T_1} x(t)\mathrm{d}t\rangle.$$

Test with a permutation

We keep $N_a = 5$, $N_z = 40$ and choose $\sigma = Id$



One periodic

We keep $N_a = 5$, $N_z = 40$ and choose $\sigma = (1\ N_z)(2\ N_z - 1)\dots$